

# Potential sensitivities to Lorentz violation from nonbirefringent modified Maxwell theory of Auger, HESS, and CTA

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## Abstract

Present and future ultra-high-energy-cosmic-ray facilities (e.g., the South and North components of the Pierre Auger Observatory) and TeV-gamma-ray telescope arrays (e.g., HESS and CTA) have the potential to set stringent bounds on the nine Lorentz-violating parameters of nonbirefringent modified Maxwell theory minimally coupled to standard Dirac theory. A concrete example is given how to obtain, in the coming decennia, two-sided bounds on the eight anisotropic parameters at the  $10^{-20}$  level and an upper (lower) bound on the single isotropic parameter at the  $+10^{-20}$  ( $-10^{-16}$ ) level. Comparison is made with existing and potential direct bounds from laboratory experiments.

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## I. INTRODUCTION

Nine real dimensionless parameters characterize nonbirefringent modified Maxwell theory minimally coupled to the standard Dirac theory of massive spin- $\frac{1}{2}$  particles [1–3]. Recently, new direct laboratory bounds on these parameters have been published [4–6], specifically, upper bounds on the absolute values of the isolated nine parameters. The direct laboratory bounds for the five parity-even anisotropic parameters are now only 1 order of magnitude above the previous indirect bounds [7, 8]. The direct laboratory bounds for the three parity-odd anisotropic parameters are still 5 orders of magnitude above the indirect bounds and the situation is even more dramatic for the single isotropic parameter [9]. (The qualifications ‘direct’ and ‘indirect’ will be explained later.)

With the laboratory bounds steadily improving [5, 10], we are, therefore, motivated to see what the chances are to tighten our indirect bounds in the coming decades. In particular, we will focus on the present and future capabilities of the Pierre Auger Observatory (PAO) [11], the High Energy Stereoscopic System (HESS) of imaging atmospheric Cherenkov telescopes [12], and the Cherenkov Telescope Array (CTA) project [13]. As regards gamma-ray telescopes, our focus is on HESS and CTA, rather than MAGIC [14] and VERITAS [15], because the first two offer the best prospects for detecting photons with the highest possible energies, which will turn out to be the crucial ingredient for one of our potential bounds. In that respect, also future extensive-air-shower arrays look promising [16].<sup>1</sup>

Returning to modified Maxwell theory, it needs to be mentioned that, in principle, the dimensionless Lorentz-violating parameters of the photon sector can be of order 1 if they arise from a nontrivial small-scale structure of spacetime [23]. The single isotropic Lorentz-violating parameter  $\tilde{\kappa}_{\text{tr}}$ , in particular, has a natural interpretation, if positive, as the excluded-spacetime-volume fraction of “defects” embedded in flat Minkowski spacetime. It is, thus, of fundamental importance to get as tight bounds as possible, preferably for all nine dimensionless Lorentz-violating parameters of the nonbirefringent photon sector but certainly for the isotropic one.

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<sup>1</sup> An entirely different class of “cosmic messengers” consists of high-energy neutrinos [17–19], which are particularly well suited for the search of new Lorentz-violating and possibly CPT-breaking effects that lie outside the mass sector [20–22].

## II. THRESHOLD CONDITIONS

An interesting suggestion for obtaining bounds on the Lorentz-violating (LV) parameters of a hypothetical nonrelativistic theory is the following [24–26]:

- (a) with modified dispersion relations, new decay channels may appear which are absent in the standard relativistic theory;
- (b) this leads to rapid energy loss of particles with energies above threshold [the threshold energy  $E_{\text{th}}$  depends on the LV parameters  $\kappa$  of the theory, typically with  $E_{\text{th}}(\kappa) \rightarrow \infty$  for  $\kappa \rightarrow 0$ ];
- (c) observing these particles implies that they necessarily have energies at or below threshold ( $E \leq E_{\text{th}}$ ), which, in turn, gives bounds on the LV parameters of the theory.

This suggestion relies, of course, on the proper calculation of the threshold energies and decay rates. See, e.g., Ref. [27] for a general discussion of physically admissible modified dispersion relations and Ref. [28] for a general discussion of Lorentz-violating decay processes, starting from simple scalar models.

Consider, now, the photon ( $\gamma$ ) of nonbirefringent modified Maxwell theory [1–3] minimally coupled to standard massive spin- $\frac{1}{2}$  Dirac particles (e.g., the electron  $e$  and proton  $p$ ) in a particular preferred frame, for definiteness, identified with the sun-centered celestial equatorial frame.<sup>2</sup> Two Lorentz-violating decay processes, in particular, have been considered: vacuum-Cherenkov radiation ( $p^+ \rightarrow p^+ \gamma$ ) [9, 29, 30] and photon decay ( $\gamma \rightarrow e^+ e^-$ ) [9, 24, 25]. We start by discussing the first process, vacuum-Cherenkov radiation of a proton  $p$  or heavy nucleus  $N$ , both, in first approximation, considered as charged pointlike Dirac particles.

Assume the detection of an ultra-high-energy-cosmic-ray (UHECR) primary with energy  $E_{\text{prim}}$ , mass  $M_{\text{prim}}$ , and flight-direction unit vector  $\hat{\mathbf{q}}_{\text{prim}}$ . In terms of the standard-model-extension parameters [3], the vacuum-Cherenkov process ( $N_{\text{prim}} \rightarrow N_{\text{prim}} \gamma$ ) then gives the following threshold condition [7, 8, 29, 30] on the LV parameters of nonbirefringent modified

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<sup>2</sup> More precisely, the theory considered is the one of the standard model with the experimentally determined values for the coupling constants and Higgs vacuum expectation value, to which a single nonbirefringent Lorentz-violating kinetic term of the  $U(1)$  gauge bosons is added, namely, the  $k_B$  term of Eq. (16) in Ref. [2] or the  $\tilde{\kappa}^{(1)}$  term of Eq. (C2a) in Ref. [9]. As remarked in Sec. V of Ref. [7], the bounds discussed in that article and this one may also apply to theories with additional Lorentz violation in the quark and lepton sectors, but, then, a precise formulation of the bounds becomes rather involved. For this reason, the discussion here is restricted to the theory with a single Lorentz-violating kinetic term of the Abelian gauge field.

Maxwell theory:

$$R[2\tilde{\kappa}_{\text{tr}} - \hat{q}_{\text{prim}}^a (\tilde{\kappa}_{\text{o}+})^{bc} \epsilon^{abc} - \hat{q}_{\text{prim}}^a (\tilde{\kappa}_{\text{e}-})^{ab} \hat{q}_{\text{prim}}^b] \leq (M_{\text{prim}} c^2 / E_{\text{prim}})^2, \quad (1)$$

where each repeated Cartesian index  $a$ ,  $b$ , and  $c$  is summed over 1 to 3, the quantity  $\epsilon^{abc}$  stands for the totally antisymmetric Levi-Civita symbol with normalization  $\epsilon^{123} = 1$ , and the ramp function  $R$  is defined by  $R[x] \equiv (x + |x|)/2$ . The velocity  $c \equiv 299\,792\,458$  m/s on the right-hand side of (1) corresponds, in the Lorentz-violating theory considered, to the maximal attainable velocity of the Dirac particles. The nine LV parameters are collected in the antisymmetric  $3 \times 3$  matrix  $\tilde{\kappa}_{\text{o}+}$  (three parameters), the symmetric traceless  $3 \times 3$  matrix  $\tilde{\kappa}_{\text{e}-}$  (five parameters), and the number  $\tilde{\kappa}_{\text{tr}}$  (one parameter).

The minimal set of UHECR events needed to obtain significant bounds from (1) on all nine LV parameters has six events (labeled  $l = 1, \dots, 6$ ) with approximately equal leverage factors  $(E_l/M_l c^2)^2 \gg 1$  and approximately (anti)orthogonal flight directions  $\hat{\mathbf{q}}_l$ . In an appropriate Cartesian coordinate system, the ideal flight directions are given by  $\hat{\mathbf{q}}_{1,2} = (\pm 1, 0, 0)$ ,  $\hat{\mathbf{q}}_{3,4} = (0, \pm 1, 0)$ , and  $\hat{\mathbf{q}}_{5,6} = (0, 0, \pm 1)$ . Strictly speaking, three orthogonal events suffice for the parity-even parameters  $\tilde{\kappa}_{\text{e}-}$  and a single event for the positive isotropic parameter  $\tilde{\kappa}_{\text{tr}}$ .<sup>3</sup> As emphasized in Ref. [25] and reiterated in Refs. [8, 9], these bounds require as only input the mere existence, at the top of the Earth's atmosphere, of charged cosmic-ray primaries with travel lengths of a meter or more. Hence, these bounds are independent of the distance to the (astronomical) source.

Next, restrict to the isotropic sector of nonbirefringent modified Maxwell theory and consider the case of a small negative parameter  $\tilde{\kappa}_{\text{tr}}$ . Then, vacuum-Cherenkov radiation is no longer possible, but another decay process becomes available, namely, photon decay into an electron-positron pair. The photon-decay process ( $\gamma \rightarrow e^+ e^-$ ) gives the following threshold condition [9, 24, 25] on the isotropic LV parameter of modified Maxwell theory:

$$R[-\tilde{\kappa}_{\text{tr}}] \leq 2 (M_e c^2 / E_\gamma)^2, \quad (2)$$

with  $E_\gamma$  the energy of the primary photon,  $M_e$  the mass of the electron, and  $R$  the ramp function defined below (1).

The minimal set of gamma-ray events needed to obtain a significant bound from (2) contains a single event with leverage factor  $(E_\gamma/M_e c^2)^2 \gg 1$  and an arbitrary flight direction

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<sup>3</sup> Remark that, with so few events, the bounds are somewhat weakened by the appearance of geometrical factors in the function argument on the left-hand side of (1). The geometrical factor of the second (parity-odd) term, for example, is of order  $\cos \pi/4 = 1/\sqrt{2} \approx 0.71$ . See also the penultimate paragraph in Sec. II of Ref. [7] for further discussion.

(the considered Lorentz violation being isotropic). Again, the only input for this bound is the mere existence, at the top of the Earth's atmosphere, of a primary photon with a travel length of a meter or more, making the bound independent of the distance to the (astronomical) source.<sup>4</sup>

### III. PROTOTYPICAL DATA SAMPLES

Two fiducial data samples are presented in this section, the first with UHECR events for application of the vacuum-Cherenkov threshold condition (1) and the second with TeV gamma rays for application of the photon-decay threshold condition (2). Our aim is to be as concrete as possible, but it is clear that the data samples considered are only examples and can certainly be modified or improved in the future, as will be discussed further in Sec. V A.

For the first prototypical data sample, we consider hybrid UHECR events [31] detected by Auger (short for the Pierre Auger Observatory, PAO). Recall that a so-called ‘hybrid’ Auger event refers to an extended air shower which has been observed simultaneously by the two types of detectors, the water-Cherenkov surface detectors and the fluorescence telescopes [11].

Specifically, the following sample  $\Sigma_{N'}$  of  $N' \equiv N/2$  hybrid Auger events is obtained from the two energy bins below  $E = 10^{19}$  eV in Figs. 2 and 3 of Ref. [31]:

$$N/2 = 131 + 96 = 227, \tag{3a}$$

$$E \sim 8 \text{ EeV}, \tag{3b}$$

$$\langle X_{\text{max}} \rangle \sim 750 \text{ g/cm}^2, \tag{3c}$$

$$\text{rms}(X_{\text{max}}) \sim 50 \text{ g/cm}^2, \tag{3d}$$

where the reason for writing  $N/2$  will become clear shortly and where  $\langle X_{\text{max}} \rangle$  stands for the average of  $X_{\text{max}}$ , the atmospheric depth of the shower maximum (here, determined by the fluorescence telescopes of Auger). The relative uncertainty of the energy measurement (3b) is taken equal to 25 % (slightly larger than the value of 22 % quoted in Ref. [31]).

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<sup>4</sup> If it could be established that certain specific astronomical sources emit UHECR protons or multi-TeV photons which definitely radiate their energy away or decay before they reach the Earth, then a finite range of nonzero values of the relevant LV parameters could be obtained, assuming the LV effects to be responsible for the observed disappearance of the particles. Such a range could be expected to have a (mild) dependence on the source distances, whereas the bounds of the present article are strictly source-distance independent.

Remark that the fluorescence telescopes of Auger [11, 31] partly track the primary particle and its secondaries along their way in the atmosphere, following them over a length of several kilometers.

The sample  $\Sigma_{N/2}$  from (3a)–(3d) is, however, not quite isotropic, as the celestial coverage of Auger–South is not complete. Awaiting results from Auger–North (or a similar cosmic-ray observatory on the northern hemisphere), we can proceed as follows. An artificial sample  $\Sigma_N$  is constructed by adding to each event of  $\Sigma_{N/2}$  one in the opposite direction:

$$\Sigma_N = \left\{ \{ (E_n, \hat{q}_n, C_n), (E_n, -\hat{q}_n, C_n) \} \mid n = 1, \dots, N/2 \right\}, \quad (3e)$$

where  $\hat{q}_n$  is the flight-direction vector of event  $n$  and  $C_n$  contains further characteristics such as  $X_{\max, n}$ . In this way, the sample  $\Sigma_N$  covers the whole celestial sphere, even though certain regions may be covered somewhat more densely than others. [There are, of course, other ways to make the sample  $\Sigma_{N/2}$  more isotropic, for example, by keeping the same number of events but randomly flipping the flight-direction vectors.]

The last two assumptions are that the sample  $\Sigma_N$  contains  $N_p \gg 1$  events with proton primaries and that the arrival directions of these  $N_p$  events are representative of an isotropic distribution. Specifically, we assume that

$$N_p \geq 48, \quad (3f)$$

and that the proton arrival directions cover the celestial sphere completely and more or less homogeneously. The isotropy condition can be described in words as follows: to have approximately  $N_p/2$  events in each hemisphere (solid angle  $2\pi$  sr), approximately  $N_p/4$  events in each quadrant (solid angle  $\pi$  sr), approximately  $N_p/8$  events in each octant (solid angle  $\pi/2$  sr), etc. Restricting to octants, one possible mathematical formulation of the isotropy condition for the proton arrival directions is

$$\forall_{n \in \{1, 2, \dots, 8\}} : N_p(\Delta\Omega_n = \pi/2) \geq 1, \quad (3g)$$

which must hold for any choice of octants  $\Delta\Omega_n$  that cover the celestial sphere ( $\Omega = 4\pi$ ) completely. Condition (3g) is not too difficult to satisfy with the relatively large total number of protons from (3f). For the data sample  $\Sigma_N$ , conditions (3f) and (3g) could perhaps result from those events with  $X_{\max} \gtrsim 825$  g/cm<sup>2</sup>, according to the calculations shown in Fig. 3 of Ref. [31] (see also Table I of Ref. [7]).

The basic idea, now, for obtaining new bounds from (1) and the prototypical data sample (3) is that, without knowing precisely which of the  $N = 454$  events would correspond to a proton, the knowledge would suffice that there would be a subset with  $N_p \gg 1$  proton events distributed isotropically. Clearly, this requires the reliable determination of the proton

fraction  $P_p$  in the sample considered (with the certainty that  $P_p > 0$ ) and the information that the protons in this UHECR sample have an overall isotropy of the arrival directions (irrespective of possible small-scale clustering). In short, the challenge for Auger will be to identify a 10–EeV UHECR sample with enough isotropic protons in it. Further discussion will be given in Sec. V A.

Turning to the second prototypical data sample, recall that 80 TeV gamma rays have been detected by HEGRA from the Crab Nebula [32] and  $10^2$  TeV gamma rays by HESS from the shell-type supernova remnant RX J1713.7–3946 [33, 34]. The relative uncertainty of the energy measurement for this last extended source equals 20 % according to Ref. [33]. Remark that the Cherenkov light detected on the ground [35] indirectly tracks the primary photon and its secondaries along their way in the atmosphere, essentially following them over a length of several kilometers.

Specifically, the second sample is taken to have a single fiducial photon event with an arbitrary arrival direction,

$$N_\gamma = 1, \tag{4a}$$

$$E_\gamma \sim 10^2 \text{ TeV}. \tag{4b}$$

Here, the crucial assumption is that at least one such photon has been detected unambiguously (or, more realistically, that this photon is one of a larger sample of photon candidates identified with high probability).

#### IV. POTENTIAL SENSITIVITIES

Relying on the proton primaries contained in the prototypical UHECR sample (3), the vacuum-Cherenkov threshold condition (1) could be used for the numerical values

$$E_{\text{prim}} \stackrel{?}{=} (8 \pm 2) \text{ EeV}, \quad M_{\text{prim}} \stackrel{?}{=} M_p = 938 \text{ MeV}, \tag{5}$$

explicitly showing the one-sigma error in the energy determination and inserting question marks to emphasize that the data sample is still artificial. Assuming a sufficiently large number  $N_p$  of proton primaries, the directions  $\hat{\mathbf{q}}_{\text{prim},l}$  could be taken optimal, so that the geometrical factors entering the argument on the left-hand side of (1) would be close to their maximal values. The main contribution to the statistical error on the bounds of the LV parameters would come from the energy uncertainty and would correspond to approximately 50 % for this sample.

The resulting two- $\sigma$  indirect bounds on the nine isolated LV parameters of nonbirefringent modified Maxwell theory would be:

$$\left| (\tilde{\kappa}_{o+})^{(ab)} \right|_{(ab)=(23), (31), (12)} \stackrel{?}{<} 1.4 \times 10^{-20}, \quad (6a)$$

$$\left| (\tilde{\kappa}_{e-})^{(ab)} \right|_{(ab)=(11), (12), (13), (22), (23)} \stackrel{?}{<} 3 \times 10^{-20}, \quad (6b)$$

$$\tilde{\kappa}_{tr} \stackrel{?}{<} 1.4 \times 10^{-20}, \quad (6c)$$

which, for the first two entries, would improve by 2 orders of magnitude and, for the last entry, by 1 order of magnitude upon the previous bounds in Ref. [8]. There would be no mystery where the improvement would come from: comparing the values (5) to those used in Ref. [8], the gain from a much smaller mass value  $M_{\text{prim}}$  would overcome the loss from a somewhat larger value of the inverse energy  $1/E_{\text{prim}}$ .

With the prototypical gamma-ray sample (4), the photon-decay threshold condition (2) could be used with numerical values

$$E_{\gamma} \stackrel{?}{=} (100 \pm 20) \text{ TeV}, \quad M_e = 0.511 \text{ MeV}. \quad (7)$$

The resulting two- $\sigma$  indirect lower bound on the single isotropic LV parameter of nonbirefringent modified Maxwell theory would be

$$\tilde{\kappa}_{tr} \stackrel{?}{>} -0.9 \times 10^{-16}, \quad (8)$$

which would improve by 1 order of magnitude upon the previous two- $\sigma$  lower bound  $\tilde{\kappa}_{tr} > -0.9 \times 10^{-15}$  [9].<sup>5</sup> Again, there would be no mystery where the improvement would come from, namely, the somewhat larger energy value  $E_{\gamma}$  in (7) compared to the one used in Ref. [9].

Both indirect bounds (6) and (8) would scale with the inverse of the energy square of the incoming particle, that is, with  $[(8 \text{ EeV})/E_{\text{prim}}]^2$  and  $[(10^2 \text{ TeV})/E_{\gamma}]^2$ , respectively. But, for the moment, these bounds are only potential bounds as they rely on certain assumptions (e.g.,  $N_p \gg 1$ ) or on low-statistics data (e.g.,  $E_{\gamma} = 10^2 \text{ TeV}$ ).

## V. DISCUSSION

### A. Experimental issues

As mentioned in the previous section, bounds (6) and (8) are, for the moment, only indicative of the potential sensitivity of Auger and HESS/CTA, because they either rely on

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<sup>5</sup> The qualitative lower bound on  $\tilde{\kappa}_{tr}$  at the  $-10^{-16}$  level (no confidence level specified) from Ref. [36] relies on a detection of 50 TeV gamma rays from the Crab Nebula, which is of marginal significance [37].



certain assumptions or use low-statistics data. We expect that both bounds will be realized in the coming decades, derived without additional assumptions and from high-statistics data. Let us now discuss the arguments on which this expectation is based.

The potential UHECR bounds (6) rely on two main assumptions for the event sample considered: (i) the large-scale isotropy of the UHECR arrival directions (applicable to all primary subspecies), and (ii) the presence of a nonvanishing fraction of proton primaries or, at least, of light-nucleus primaries. Both assumptions will be tested (and, most likely, verified) with further Auger–South data and forthcoming Auger–North data.

At this point, it is already clear that it will be of critical importance for Auger to obtain, for a subset of events, a reliable determination of the type of primary (e.g., a hydrogen, helium, oxygen, or iron nucleus), preferably by use of different types of diagnostics (for example, in addition to  $X_{\max}$ , number densities of muons and arrival times of particles reaching the surface [38]). As mentioned in Sec. II, only six (anti)orthogonal events with  $(M_{\text{prim}} c^2/E_{\text{prim}})^2 \sim 10^{-20}$  would suffice, as long as  $M_{\text{prim}}$  would be determined accurately.

Alternatively, a nonzero fraction  $P_i$  of primary type  $i$  (for example,  $i = p$ , He, O, or Fe) would suffice for establishing the same bound, provided a large enough sample of isotropic events would be available with  $(M_i c^2/E_i)^2 \sim 10^{-20}$  [this is, in fact, the strategy followed with the prototypical data sample (3) used to obtain the potential bounds (6) for  $i = p$  and  $M_i = 0.938$  GeV]. This strategy, with a nonzero  $P_i$  for an appropriate sample of UHECR events, appears the most promising for improving the bounds of Ref. [8].

Recent observations from Auger [31] indicate an increasing fraction of heavy nuclei with increasing primary energy  $E$ . Still, it is very well possible that the fraction of light nuclei (e.g., hydrogen) is small but nonzero at any fixed value of  $E$ . In fact, a recent suggestion [39] to explain the rising fraction of ironlike nuclei seems to imply just that (e.g.,  $P_p \sim 10\%$  and  $P_{\text{Fe}} \sim 90\%$  at  $E \sim 8$  EeV for the simple model of Fig. 1 of Ref. [39]). It, then, remains to determine the optimal primary type  $i$  with the smallest value of  $M_i c^2/E_i$  and enough events ( $P_i N \gtrsim 10$ ) having arrival directions more or less isotropically distributed over the celestial sphere (irrespective of the positions of the original UHECR sources). As mentioned already in Sec. III, the argument for obtaining the bound is statistical, without the need to know precisely which event of the  $N$ –event sample corresponds to species type  $i$ , as long as we can be sure about the nonzero value of the fraction  $P_i$  and the arrival-direction isotropy.

Now turn to the potential gamma-ray bound (8), which relies on the detection of  $10^2$  TeV gamma rays by HESS. For the moment, this detection has only a marginal significance (less than two- $\sigma$  according to Fig. 3 and Table 5 of Ref. [33]). Future improvement can be expected from further HESS (or VERITAS) observations and certainly with CTA [13],

which may have a 10 times greater sensitivity than HESS at energies  $E_\gamma \sim 10^2$  TeV. If CTA (or an extensive-air-shower array [16]) unambiguously detects  $10^2$  TeV photons from whichever astronomical source, the potential bound (8) will be transformed into one of the cleanest bounds on isotropic Lorentz violation in the photon sector.

In addition, there is always the possibility of a detection by Auger of ultra-high-energy photons [40, 41]. Note that a photon event with  $E_\gamma = 10^{18}$  eV, for example, would improve the lower bound (8), based on (2) and (7), by a factor  $10^8$  to the level of  $-0.9 \times 10^{-24}$ .

## B. Theoretical issues

Following up on these experimental considerations, we have two remarks on theoretical issues, one regarding the distinction of ‘direct’ vs. ‘indirect’ bounds already made in Sec. I and the other regarding the distinction of ‘earth-based’ vs. ‘astrophysical’ bounds.

First, a direct bound on the LV parameters of nonbirefringent modified Maxwell theory [1–3] is taken to refer to experiments which directly test the modified propagation properties of the nonstandard photons, for example, by measuring positions and arrival times of pulses of light. An indirect bound, instead, is taken to refer to experiments which look for derived effects resulting from the nonstandard dispersion relations of the photons. These derived effects [here, vacuum-Cherenkov radiation and photon decay] are, however, unambiguously calculable from the theory considered in this article, which has a modified pure-photon sector and a standard matter sector with minimal photon-matter couplings (governed by the principle of gauge invariance). In fact, the energy thresholds used for the indirect bounds of Refs. [8, 9] and the present article only rely on energy-momentum conservation. For this reason, we do not consider indirect bounds to be necessarily less reliable than direct bounds.

Second, it is important to realize that the existing indirect bounds from Refs. [8, 9] and the potential indirect bounds from the present article are earth-based bounds (the atmosphere of the Earth corresponding to the ‘front end’ of the detector [11, 12]) and not astrophysical bounds, which may rely on additional assumptions about the astronomical source and the propagation distance of the primary particles. We have already mentioned in Secs. II and III that our indirect LV bounds require the existence of primaries with track lengths of a meter and that Auger measures shower tracks which start at a point high up in the Earth’s atmosphere and run for several kilometers. Hence, we can simply repeat what was written in Ref. [8], “these Cherenkov bounds only depend on the measured energies and flight directions of the charged cosmic-ray primaries at the top of the Earth atmosphere.” The same holds

for the photon-decay bound, now referring to measurements of the photon primary. As such, these earth-based bounds are not inherently inferior to laboratory bounds.

### C. Comparison of bounds

Table I now compares the different types of bounds for the parameters of nonbirefringent modified Maxwell theory minimally coupled to standard Dirac theory. The second data row of this table refers to a potential sensitivity of order  $10^{-20}$  [5] which may be achieved with cryogenic resonators [10]. As it stands, direct laboratory experiments are affected by the earth-velocity boost factor  $\beta_{\oplus} \sim 10^{-4}$ , which reduces the sensitivity for the parity-odd parameters  $\tilde{\kappa}_{o+}$  linearly and for the isotropic parameter  $\tilde{\kappa}_{tr}$  quadratically. There has been a suggestion [42] of how, in principle, this boost-dependence problem may be overcome for the parity-odd parameters, but, to the best of our knowledge, this has not yet been realized experimentally.

As the effects of the Earth's velocity are negligible for the indirect bounds from vacuum-Cherenkov radiation and photon decay [7–9], the corresponding indirect bounds for the parity-odd and isotropic parameters are much stronger than the direct bounds.<sup>6</sup> Note also that it is entirely possible that future data samples from Auger–South+North and CTA may provide us with 3 times larger energies than used in (5) and (7), so that the sensitivities of the last data row in Table I would be improved by an order of magnitude to the level of  $10^{-21}$  for the eight anisotropic parameters<sup>7</sup> and  $(-10^{-17}, +10^{-21})$  for the single isotropic parameter. Especially, the latter projected CTA/Auger sensitivities for the isotropic parameter  $\tilde{\kappa}_{tr}$  would be a welcome complement to future laboratory bounds.

## VI. CONCLUSION

The current direct laboratory bounds [4–6] on the nine Lorentz-violating parameters of nonbirefringent modified Maxwell theory [1–3] minimally coupled to standard Dirac theory are absolutely remarkable. Still, they fall short of what has been achieved indirectly [8, 9],

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<sup>6</sup> The same holds for an indirect laboratory bound (relying on derived effects for the synchrotron radiation rate at LEP) which gives  $|\tilde{\kappa}_{tr}| < 5 \times 10^{-15}$  at the two- $\sigma$  level [43].

<sup>7</sup> Inferred two- $\sigma$  bounds on the eight anisotropic parameters already exist at the  $10^{-20}$  level (even better for the parity-even parameters) from an atomic-fountain-clock laboratory experiment [44], if a coordinate transformation is used to move the isolated Lorentz violation in the proton sector (eight parameters) to the electromagnetic sector (eight parameters). For further discussion of these coordinate transformations, see, e.g., App. A in Ref. [45] and Sec. VII in Ref. [46].

TABLE I: Orders of magnitude for existing and potential two- $\sigma$  bounds on the nine Lorentz-violating parameters of nonbirefringent modified Maxwell theory [1–3] (minimally coupled to standard Dirac theory) from direct laboratory and indirect earth-based experiments. The considered indirect bounds for the isotropic parameter  $\tilde{\kappa}_{\text{tr}}$  have different orders of magnitude depending on the sign of  $\tilde{\kappa}_{\text{tr}}$  [the values of the last data row, for example, correspond to (8) and (6c) in the main text]. The second data row for potential direct laboratory bounds shows explicitly the effects from the earth-velocity boost factor  $\beta_{\oplus} \sim 10^{-4}$ .

Type of bound	$\tilde{\kappa}_{\text{e-}}$	$\tilde{\kappa}_{\text{o+}}$	$\tilde{\kappa}_{\text{tr}}$	Reference
Existing, direct	$10^{-17}$	$10^{-13}$	$10^{-8}$	[4–6]
Potential, direct	$10^{-20} ?$	$10^{-20+4} ?$	$10^{-20+8} ?$	[5, 10]
Existing, indirect	$10^{-18}$	$10^{-18}$	$(-10^{-15}, +10^{-19})$	[8, 9]
Potential, indirect	$10^{-20} ?$	$10^{-20} ?$	$(-10^{-16} ?, +10^{-20} ?)$	This paper

as can be seen by comparing the first and third data rows in Table I. Instead of arguing about the relative merits of these direct and indirect bounds (cf. Sec. VB), we adopt the following pragmatic approach.

The existing indirect earth-based bounds of Refs. [8, 9] can be used to predict that future direct or indirect laboratory experiments will not detect nonbirefringent Lorentz violation in the photon sector at levels of order  $10^{-18}$  for the eight anisotropic parameters and  $10^{-15}$  for the single isotropic parameter. Furthermore, this article has shown that experiments at present and future observatories such as Auger and CTA have the potential to reduce these predictions to the  $10^{-20}$  level for the anisotropic parameters and to the  $10^{-16}$  level for the isotropic parameter (see the last data row in Table I). Perhaps the most interesting predictions of Auger and CTA would be for the isotropic parameter  $\tilde{\kappa}_{\text{tr}}$ , which might ultimately drop to levels of order  $(-10^{-17}, +10^{-21})$  for energy values  $E_{\text{prim}} = 25 \text{ EeV}$  and  $E_{\gamma} = 3 \times 10^2 \text{ TeV}$  replacing those of Eqs. (5) and (7), respectively.

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